

GOLDSTONE FERMION DARK MATTER

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Flip Tanedo

Cornell  University

In collaboration with B. Bellazzini, C. Csáki, J. Hubisz, and J. Shao
SUSY, 29 August 2011

The WIMP Miracle

Contains factors of $M_{\text{Pl}}, s_0, \dots$

$$\Omega_{\text{DM}} h^2 \approx 0.1 \left(\frac{x_f}{20} \right) \left(\frac{g_*}{80} \right)^{-\frac{1}{2}} \left(\frac{\langle \sigma v \rangle_0}{3 \times 10^{-26} \text{ cm}^3/\text{s}} \right)$$

$$\sim \left\langle \frac{\alpha^2 v}{(100 \text{ GeV})^2} \right\rangle$$

Within orders of magnitude!

Ωh^2 vs direct detection

Tension between annihilation cross section and direct detection bounds

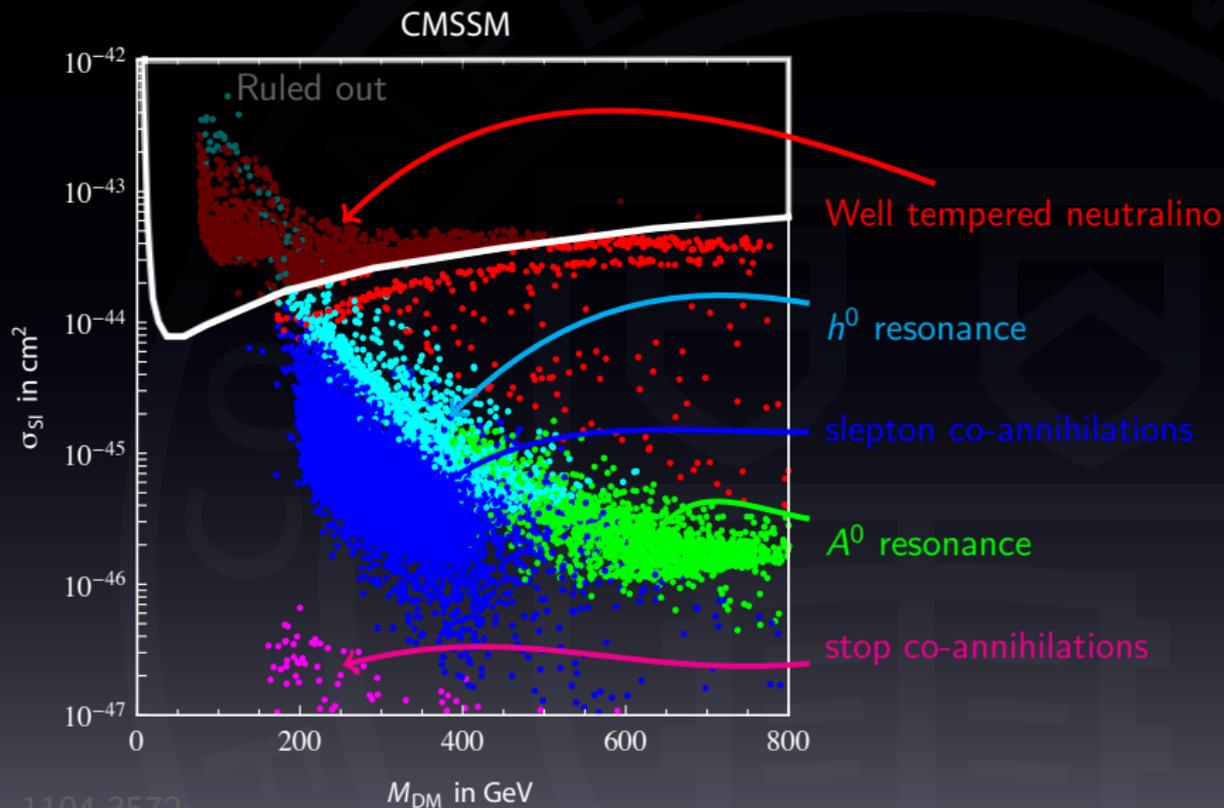
$$\sigma_{\text{ann.}} \sim 0.1 \text{ pb}$$

$$\sigma_{\text{SI}} \sim 7.0 \times 10^{-9} \text{ pb}$$

50 GeV WIMP

Typical strategy: pick parameters such that σ_{SI} is **suppressed**, then use tricks to **enhance** $\sigma_{\text{ann.}}$.

Ωh^2 vs direct detection



1104.3572

Motivation I: a natural WIMP

Typical MSSM WIMP: σ_{SI} **too large**

Want to naturally suppress direct detection while maintaining 'miracle' of successful abundance.

If LSP is part of a **Goldstone multiplet**, $(s + ia, \chi)$, additional suppression from derivative coupling.

- Like a weak scale axino, but unrelated to CP
- Like singlino DM, but global symmetry broken in SUSY limit

Motivation I: a natural WIMP

Annihilation: p -wave decay to Goldstones

$$\frac{1}{f} \bar{\chi} \gamma^\mu \gamma^5 \chi \partial_\mu a \quad \Rightarrow \quad \langle \sigma v \rangle \approx \left(\frac{m_\chi}{f^2} \right)^2 \left(\frac{T_f}{m_\chi} \right) \approx 1 \text{ pb}$$

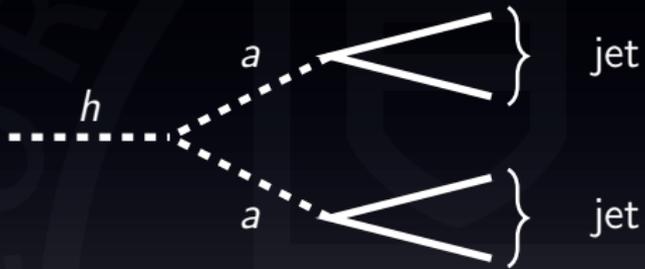
Direct detection: CP-even Goldstone mixing with Higgs

$$\frac{m_\chi v}{f^2} \sim 0.01 \quad \Rightarrow \quad \sigma_{\text{SI}} = \left(\frac{m_\chi v}{f^2} \right)^2 \sigma_{\text{SI}}^{\text{MSSM}} \approx \mathcal{O}(10^{-45} \text{ cm}^2)$$

Motivation II: Buried Higgs

Idea: Light Higgs buried in QCD background

Global symmetry at $f \sim 500$ GeV with coupling $\frac{1}{f^2} h^2 (\partial a)^2$



0906.3026, 1012.1316, 1012.1347

Can we **bury** the Higgs through a decays,
but **dig up** dark matter in χ ?



The Goldstone Supermultiplet

sGoldstone

Goldstone boson

Goldstone fermion

$$A = \frac{1}{\sqrt{2}} (s + i a) + \sqrt{2}\theta \chi + \theta^2 F$$

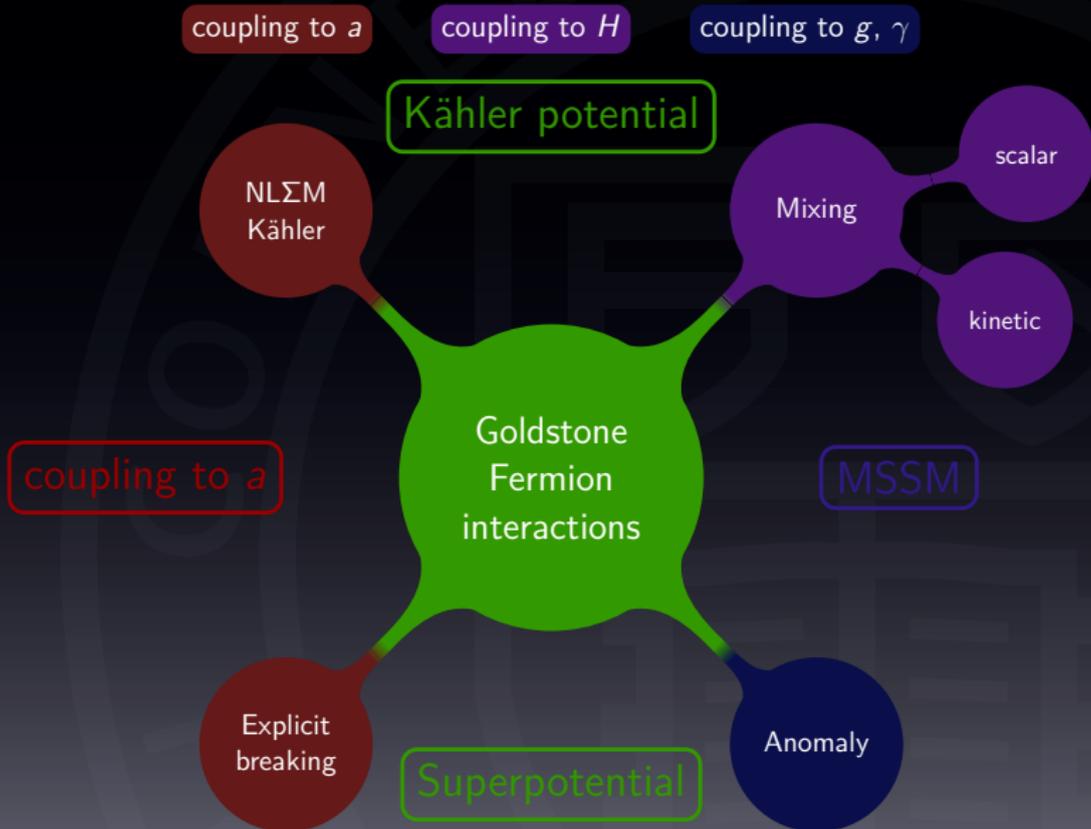
Carries the low-energy degrees of freedom of the UV fields,

$$\Phi_i = f_i e^{q_i A/f}$$

$$f^2 = \sum_i q_i^2 f_i^2$$

$SUSY \Rightarrow$ explicit s mass, $m_\chi \approx q_i \langle F_i \rangle / f$, a massless

Interactions: Overview



Interactions: $NL\Sigma M$ Kähler potential

Non-linear realization of the global $U(1)$

\Rightarrow Kähler interactions of the Goldstone multiplet:

$$\frac{\partial^2 K}{\partial A \partial A^\dagger} = 1 + b_1 \frac{q}{f} (A + A^\dagger) + \dots \quad b_1 = \frac{1}{qf^2} \sum_i q_i^3 f_i^2$$

Note $K = K(s)$, manifest shift-invariance.

$$\mathcal{L} = \left(1 + b_1 \frac{\sqrt{2}}{f} s + \dots \right) \left(\frac{1}{2} (\partial s)^2 + \frac{1}{2} (\partial a)^2 + \frac{i}{2} \bar{\chi} \gamma^\mu \partial_\mu \chi \right) + \frac{1}{2\sqrt{2}} \left(b_1 \frac{1}{f} + b_2 \frac{\sqrt{2}}{f^2} s + \dots \right) (\bar{\chi} \gamma^\mu \gamma^5 \chi) \partial_\mu a + \dots$$

Phys. Lett. B 87 (1979) 203

b_1 controls the annihilation cross section.

Interactions: scalar mixing

MSSM fields are uncharged under the global U(1), but may mix with the Goldstone multiplet through higher-order terms in K :

$$K = \frac{1}{f} (A + A^\dagger) (c_1 H_u H_d + \dots) + \frac{1}{2f^2} (A + A^\dagger)^2 (c_2 H_u H_d + \dots)$$

The new scalar interactions take the form

$$\mathcal{L} \supset \left[\frac{1}{2} (\partial a)^2 + \frac{1}{2} \bar{\chi} \not{\partial} \chi \right] \left(1 + c_h \frac{v}{f} h + \dots \right)$$

c_h depends on c_i and the Higgs mixing angles.

c_h controls direct detection

Interactions: kinetic mixing

The higher order terms in K also induce kinetic $\tilde{H}-\chi$ mixing.

$$\mathcal{L} \supset i\epsilon_u \bar{\chi} \gamma^\mu \partial_\mu \tilde{H}_u^0 + i\epsilon_d \bar{\chi} \gamma^\mu \partial_\mu \tilde{H}_d^0 + \text{h.c.}$$

where $\epsilon \sim v/f$. For large μ , χ has a small \tilde{H} of order $v m_\chi / f \mu$.

Mixing with other MSSM fields is suppressed. Assuming MFV,

$$K = \frac{1}{f} (A + A^\dagger) \left(\frac{Y_u}{M_u} \bar{Q} H_u U + \dots \right)$$

where the scales $M_{u,d,\ell}$ are unrelated to f or v and can be large and dependent on the UV completion

Interactions: anomaly

Fermions Ψ charged under global U(1) and Standard Model

$$\mathcal{L}_{\text{an}} \supset \frac{c_{\text{an}}}{f\sqrt{2}} \left(a G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a + 2\bar{\chi} G_{\mu\nu}^a \sigma^{\mu\nu} \gamma^5 \lambda^a \right)$$

$$c_{\text{an}} = \frac{\alpha}{8\pi} \sqrt{2} \sum_i^{N_\Psi} \left(\frac{y_i f}{m_{\Psi_i}} \right) = \frac{\alpha}{8\pi} q_\Psi N_\Psi$$



Also $\gamma\gamma$

Assumed: degenerate m_Ψ and $y = m_\Psi q_\Psi / f\sqrt{2}$

Integrating out λ^a generates χ couplings to gluons

$$\mathcal{L} \supset - \left(\frac{c_{\text{an}}^2}{2M_\chi f^2} \right) \bar{\chi} \chi G G - i \left(\frac{c_{\text{an}}^2}{2M_\chi f^2} \right) \bar{\chi} \gamma^5 \chi G \tilde{G}$$

These contribute to collider and astro operators.

Interactions: explicit breaking

Include explicit $U(1)$ spurion $R_\alpha = \lambda_\alpha f$ with $\lambda_\alpha \ll 1$

$$W_{U(1)} = f^2 \sum_\alpha R_{-\alpha} e^{aA/f}$$

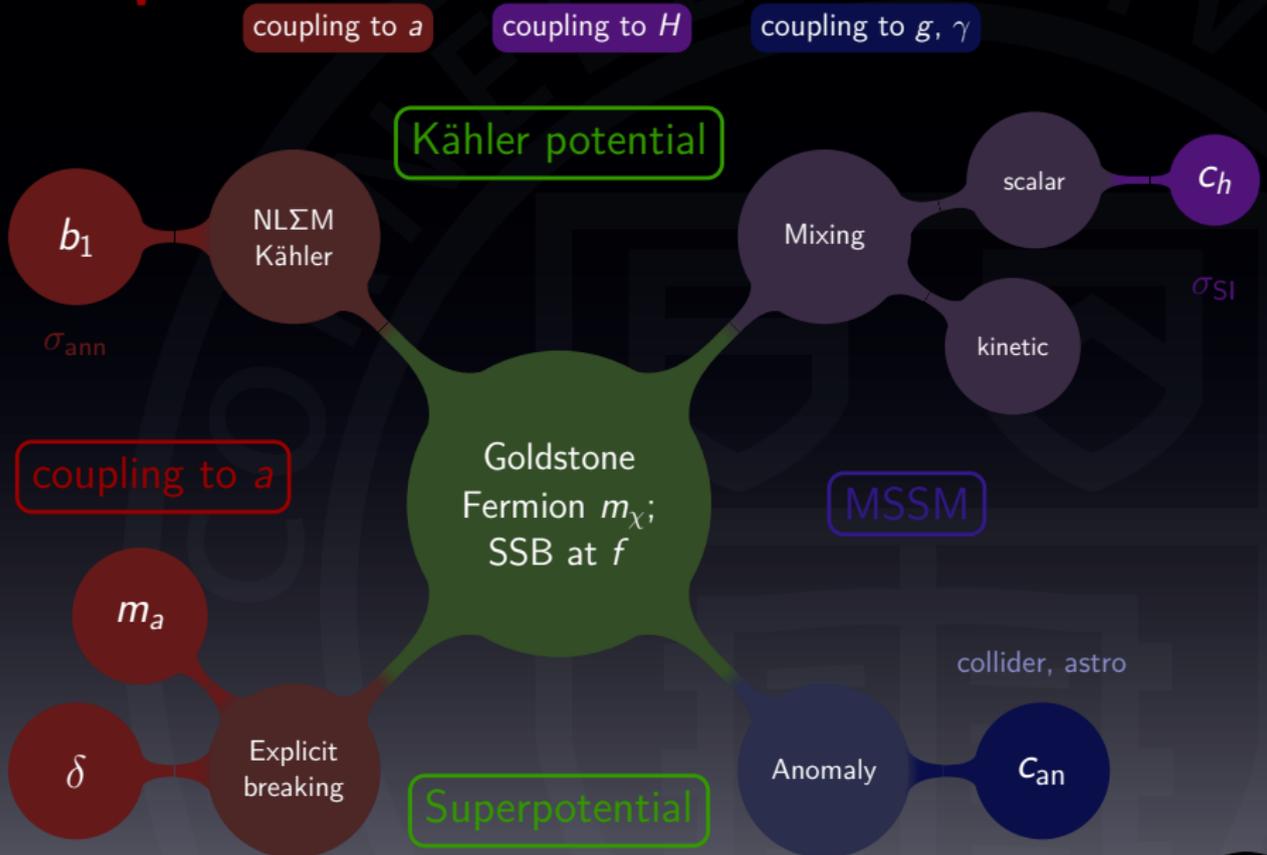
Preserve SUSY \Rightarrow at least two spurions with opposite charge.

This generates $m_a = m_\chi = m_s$ and couplings

$$\mathcal{L} \supset - \underbrace{\frac{m_a}{2\sqrt{2}f}(\alpha + \beta)}_\delta i a \bar{\chi} \gamma^5 \chi + \underbrace{\frac{m_a}{8f^2}(\alpha^2 + \alpha\beta + \beta^2)}_\rho a^2 \bar{\chi} \chi$$

By integration by parts this is equivalent to a shift in the b_1 coefficient from the Kähler potential

Main parameters



Parameter space scan

Abundance: $\langle \sigma v \rangle \approx \frac{b_1^4}{8\pi} \frac{T_f}{m_\chi} \frac{m_\chi^2}{f^4} \approx 1 \text{ pb}$

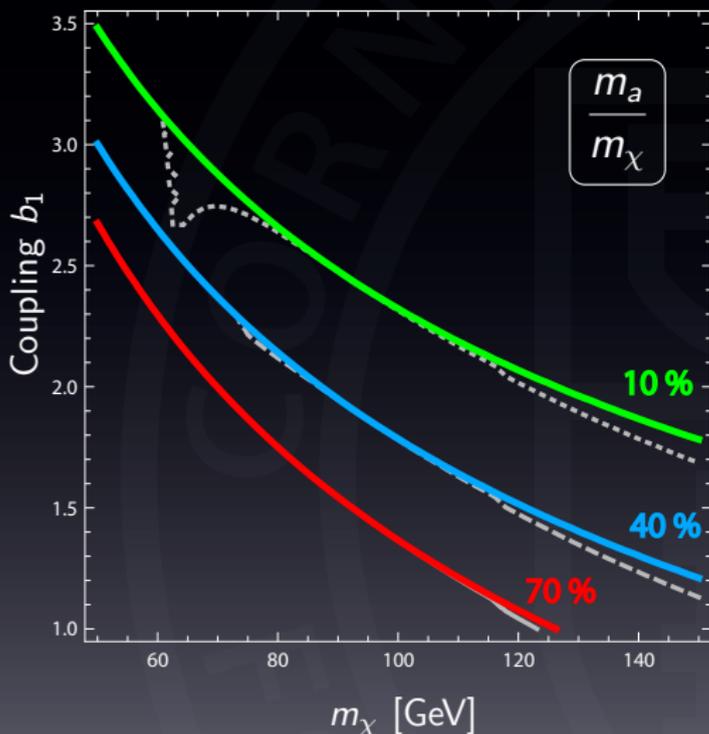
p -wave: $b_1 \gtrsim 1$, all other parameters take natural values

Parameter	Description	Scan Range
f	Global symmetry breaking scale	500 GeV – 1.2 TeV
m_χ	Goldstone fermion mass (SUSY)	50 – 150 GeV
m_a	Goldstone boson mass	8 GeV – $f/10$
b_1	$\chi\chi a$ coupling	[0, 2]
c_{an}	Anomaly coefficient	0.06
c_h	Higgs coupling	[-1, 1]
δ	Explicit breaking $ia\bar{\chi}\gamma^5\chi$ coupling	3/2

$$\mathcal{L} \supset \left[\frac{1}{2}(\partial a)^2 + \frac{1}{2}\bar{\chi}\not{\partial}\chi \right] c_h \frac{v}{f} h + \frac{b_1}{2\sqrt{2}f} (\bar{\chi}\gamma^\mu\gamma^5\chi) \partial_\mu a + \frac{c_{an}}{f\sqrt{2}} a G\tilde{G} + i\delta a\bar{\chi}\gamma^5\chi$$

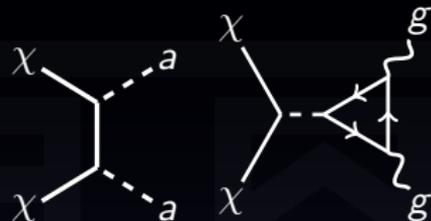
Contours of fixed Ω

$$\Omega h^2 = 0.11$$



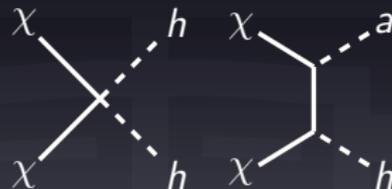
Dominant contribution

Kähler, anomaly, $U(1)$



Subleading

Mixing with Higgs



Negligible

$\chi\chi \rightarrow s \rightarrow aa$, $\chi\chi \xrightarrow{t,u} hh$

Direct Detection

Higgs exchange typically dominates by a factor of $\mathcal{O}(10^3)$.

$$\sigma_{\text{SI}}^{\text{H}} \approx 3 \cdot 10^{-45} \text{ cm}^2 c_h^2 \left(\frac{115 \text{ GeV}}{m_h} \cdot \frac{700 \text{ GeV}}{f} \right)^4 \left(\frac{m_\chi}{100 \text{ GeV}} \cdot \frac{\mu_\chi}{\text{GeV}} \cdot \frac{\lambda_N}{0.5} \right)^2$$

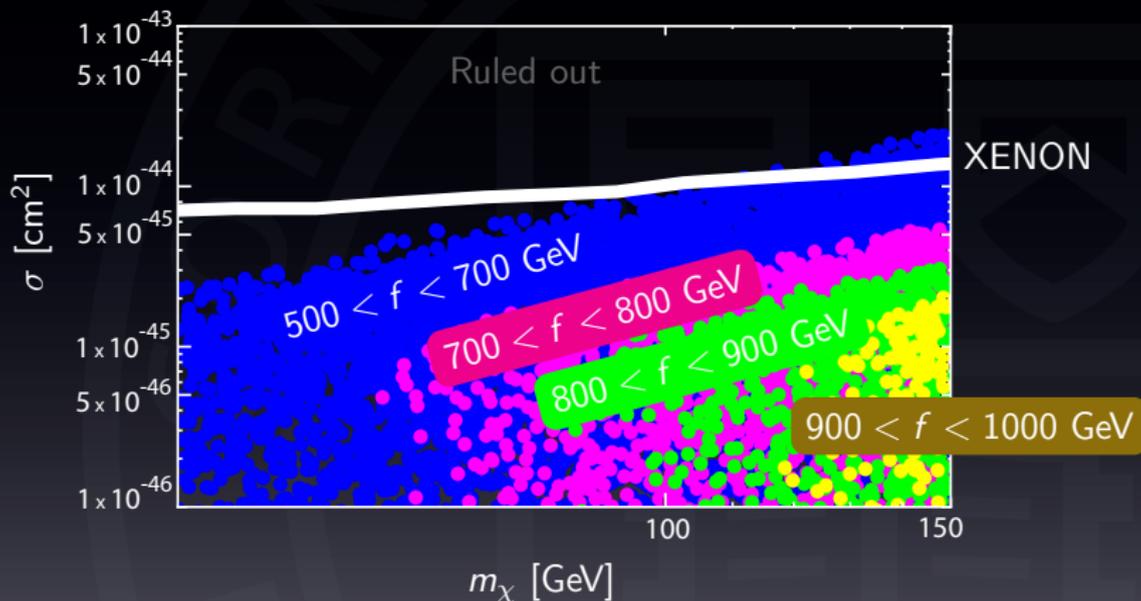
Compare this to the MSSM Higgs with $\mathcal{L} = \frac{1}{2}cg\bar{\chi}\chi h$:

$$\sigma_{\text{SI}}^{\text{MSSM}} \sim \frac{c^2 g^2 \lambda_N^2 \mu^2 m_N^2}{2\pi m_h^2 v^2} \approx c^2 \times 10^{-42} \text{ cm}^2$$

Natural suppression: $(m_\chi v/f^2)^2$ due to Goldstone nature

Parameter space scan

Direct Detection



Indirect detection & Colliders

\bar{p} spectrum: below PAMELA

- (Einsasto DM Halo profile) 1104.3572

γ -ray line search: $\mathcal{O}(10)$ smaller than bound

- $\chi\chi \rightarrow a \rightarrow \gamma\gamma$

Diffuse γ -ray spectrum: $\mathcal{O}(10)$ smaller than bound

- $\chi\chi \rightarrow a \rightarrow gg \rightarrow \pi's$

Photo-production from annihilation: σ 3x lower than bound

- Low mass DM $m_\chi \lesssim 60$ GeV, constrains bb decays

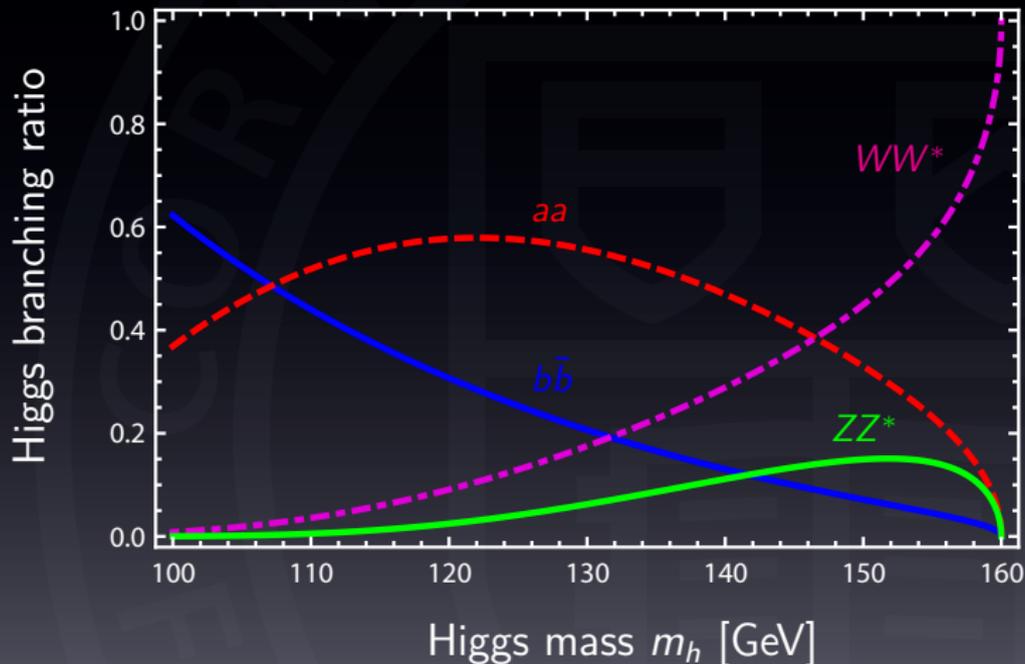
ISR monojets at colliders: dim-7 operators too small

$$\mathcal{L} \supset -\frac{C_{an}^2}{2M_\lambda f^2} \bar{\chi}\chi GG - \frac{iC_{an}^2}{2M_\lambda f^2} \bar{\chi}\gamma^5\chi G\tilde{G}$$

Non-standard Higgs decays

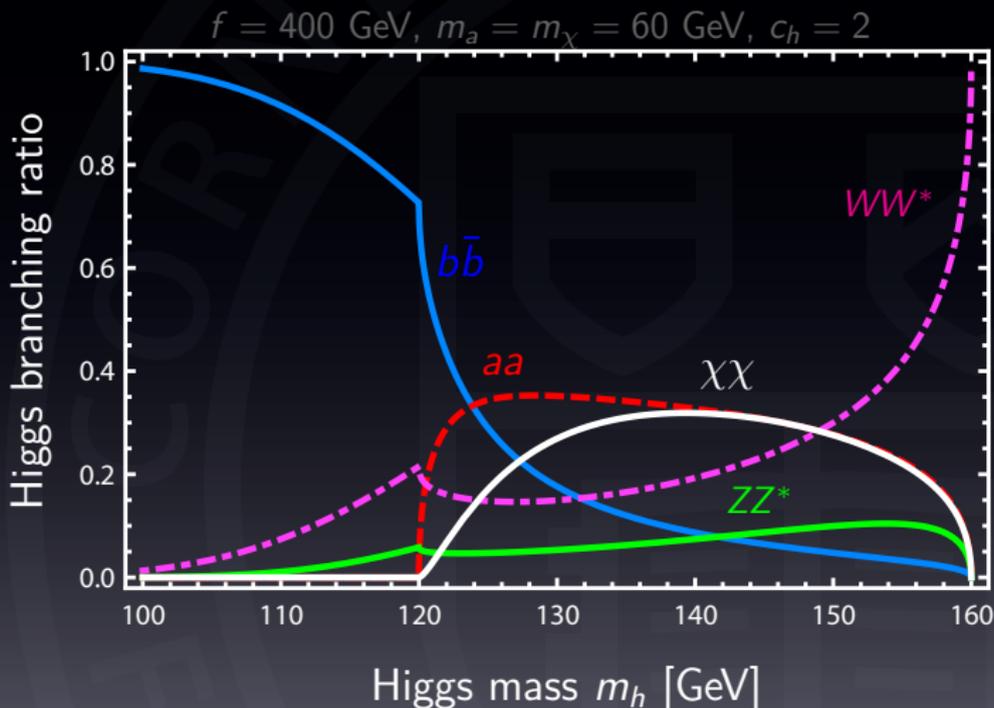
Hard to completely bury the Higgs. LEP: $\text{Br}(\text{SM}) \gtrsim 20\% \Rightarrow m_h \gtrsim 110 \text{ GeV}$

$f = 500 \text{ GeV}, m_a = 45 \text{ GeV}, m_\chi = 100 \text{ GeV}, c_h = 2$



Non-standard Higgs decays

Partially buried & invisible: Suppressed SM channels, MET, $\Gamma_{\text{tot}} < 1$



Conclusions

Executive summary: Goldstone Fermion dark matter

- SSB: global $U(1) \Rightarrow$ Goldstone boson a and fermion χ
- χ is LSP and DM, a gives 'buried' Higgs channel

Simple extension of MSSM with natural WIMP dark matter

- Kähler $\chi\chi a$ interaction controls abundance
- Higgs mixing, anomaly controls direct detection
- Novel collider signature: partially buried/invisible Higgs

Further directions:

- p -wave Sommerfeld enhancement (can push m_a, m_χ to 10 GeV)
- Non-abelian generalization

Extra Slides

Examples of Linear Models

Simplest example:

$$W = yS \left(\bar{N}N - \mu^2 \right) + \underbrace{N\bar{\phi}\phi}_{\text{anomaly}} + \underbrace{SH_uH_d}_{\text{mixing}} + \underbrace{W_{\text{explicit}}}_{\text{explicit } U(1)}$$

Example with $|b_1| \geq 1$:

$$W = \lambda XYZ - \mu^2 Z + \frac{\tilde{\lambda}}{2} Y^2 N - \tilde{\mu} \bar{N}N$$

$q_Z = 0$, $q_N = -q_{\bar{N}} = -2q_Y = 2q_X$. Goldstone multiplet:

$$A = \sum_i \frac{q_i f_i \psi_i}{f} = \frac{q_Y}{f} \left(Y f_Y - X f_X + 2 \bar{N} F_{\bar{N}} \right)$$

$$b_1 = \frac{-f_X^2 + f_Y^2 + 8f_N^2}{f_X^2 + f_Y^2 + 4f_N^2}$$

Tamvakis-Wyler Theorem

Phys. Lett B 112 (1982) 451; Phys. Rev. D 33 (1986) 1762

Global symmetry: $W[\Phi_i] = W[e^{i\alpha q_i} \Phi_i]$ so that

$$0 = \frac{\partial W[e^{i\alpha q_i} \Phi_i]}{\partial \alpha} = \sum_j W_j q_j \Phi_j,$$

Taking a derivative $\partial/\partial\Phi_i$ gives:

$$0 = \frac{\partial}{\partial \Phi_i} \left(\sum_j W_j q_j \Phi_j \right) \Big|_{\langle \Phi \rangle} = \sum_j W_{ij} q_j f_j + W_i q_i$$

SUSY NL Σ M

Phys. Lett. B 87 (1979) 203

Expand Kähler potential, drop total derivatives, integrate out F :

$$\begin{aligned}\mathcal{L} = & K'' \left(\frac{i}{2} \partial\chi\sigma\bar{\chi} + |\partial\phi|^2 \right) \\ & + \frac{K'''}{4} i\chi\sigma\bar{\chi}\partial(\phi - \phi^*) \\ & + \frac{1}{4} \left(K'''' - \frac{(K''')^2}{K''} \right) \chi^2\bar{\chi}^2\end{aligned}$$

These terms can be understood in terms of geometric properties of the vacuum manifold, see e.g. [hep-th/0101055](https://arxiv.org/abs/hep-th/0101055).

SUSY Breaking and χ mass

We assume that soft SUSY terms that also explicitly break the global U(1) are negligible. Neglect D -term mixing with λ^a , then fermion mass matrix is W_{ij} . Tamvakis-Wyler:

$$\sum_j W_{ij} q_j f_j = -q_i W_i = -q_i F_i$$

so that $\chi = \sum_i q_i f_i \psi_i / f$ mass depends on how U(1)-charged F -terms in the presence of soft SUSY terms.

If W has an unbroken R symmetry, then $R[\chi] = -1$ which prohibits a Majorana mass. However, while soft scalar masses preserve R , A -terms are holomorphic and generally break R symmetries to contribute to m_χ .

SUSY Breaking and χ mass

The A -term contribution to m_χ is equivalent to F -term mixing between $U(1)$ charged fields and the SUSY spurion, X . This was recently emphasized in 1104.0692 as an irreducible $\mathcal{O}(m_{3/2})$ contribution to the Goldstone fermion

For concreteness, consider gravity mediation with $m_{\text{soft}} \sim F/M_{\text{Pl}}$.

$$K = \sum_i Z(X, X^\dagger) \phi_i^\dagger \phi_i$$

Analytically continue into superspace [hep-ph/9706540](https://arxiv.org/abs/hep-ph/9706540)

$$\phi \rightarrow \phi' \equiv Z^{1/2} \left(1 + \frac{\partial \ln Z}{\partial X} F \theta^2 \right) \phi$$

Canonical normalization generates A -terms:

$$\Delta \mathcal{L}_{\text{soft}} = \left. \frac{\partial W}{\partial \Phi} \right|_{\Phi=\phi} Z^{-1/2} \left(- \frac{\partial \ln Z}{\partial \ln X} \frac{F}{M} \right)$$

SUSY Breaking and χ mass

$$\Delta\mathcal{L}_{\text{soft}} = \left. \frac{\partial W}{\partial \Phi} \right|_{\Phi=\phi} Z^{-1/2} \left(-\frac{\partial \ln Z}{\partial \ln X} \frac{F}{M} \right)$$

Completely incorporates F -term mixing of the form $FF_i^\dagger \Phi_i$. The χ mass is determined by the induced F_i obtained by minimizing

$$V = \left| \frac{\partial W}{\partial \phi_i} \right|^2 + A_i \frac{\partial W}{\partial \phi_i} \phi_i + \text{h.c.} + m_i^2 |\phi_i|^2$$

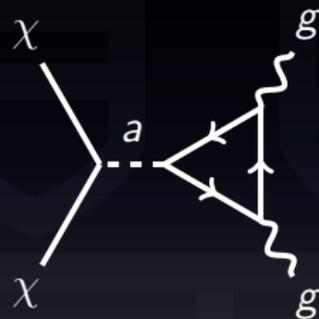
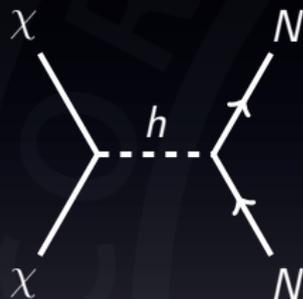
Assuming $A_i, m_i < f_i$, generic size is $|F_i| \approx A_i f_i$ so that $m_\chi \sim A_i q_i$. Often the A -terms are suppressed relative to other soft terms, so it's reasonable to expect χ to be the LSP.

Contributions from soft scalar masses are on the order of m_i^2/f_i which can easily be suppressed.

Direct Detection

Relevant couplings from EWSB and anomaly:

$$\mathcal{L} \supset \frac{c_h^V}{2f} \bar{\chi} \not{\partial} \chi h - \frac{c_{an}^2}{2M_\lambda f^2} \bar{\chi} \chi GG - \frac{i c_{an}^2}{2M_\lambda f^2} \bar{\chi} \gamma^5 \chi G \tilde{G}$$



Effective coupling to nucleons: $\mathcal{L} = G_{\text{nuc}} \bar{N} N \bar{\chi} \chi$,

$$G_{\text{nuc}} = c_h \frac{\lambda_N}{2\sqrt{2}} \left(\frac{m_\chi m_N}{m_h^2 f^2} \right) + \frac{4\pi c_{an}^2}{9\alpha_s} \frac{m_N}{M_\lambda f} \left(1 - \sum_{i=u,d,s} f_i^{(N)} \right)$$

Direct detection: nucleon matrix elements

Nucleon matrix elements can be parameterized via

Phys. Rev. D38 2869, Phys. Lett. B219 347, 0801.3656, 0907.417

$$m_i \langle N | \bar{q}_i q_i | N \rangle = f_i^{(N)} m_N$$

The heavy quark contribution via gluons can be calculated by the conformal anomaly, Phys. Lett. B78 433

$$f_j^{(N)} m_N = \frac{2}{27} \left(1 = \sum_{q=u,d,s} f_q^{(N)} \right) \quad j = c, b, t$$

Relevant quantity in Higgs exchange: c_q , diagonalized Yukawa

$$\lambda_N = \sum_{q=u,d,s} c_q f_q^{(N)} + \frac{2}{27} \left(1 = \sum_{q=u,d,s} f_q^{(N)} \right) \sum_{q'=c,b,t} c_{q'}$$

Direct Detection

Some details:

$$G_{\chi N} = c_h \frac{\lambda_N}{2\sqrt{2}} \left(\frac{m_\chi m_N}{m_h^2 f^2} \right) + \frac{4\pi c_{an}^2}{9\alpha_s} \frac{m_N}{M_\lambda f} \left(1 - \sum_{i=u,d,s} f_i^{(N)} \right)$$

For reduced mass $\mu_\chi = (m_\chi^{-1} + m_N^{-1})^{-1}$,

$$\sigma_{\text{SI}}^{\text{Higgs}} = \frac{4\mu_\chi^2}{A^2\pi} [G_{\chi p} Z + G_{\chi n} (A - Z)]$$

$$\sigma_{\text{SI}}^{\text{H}} \approx 3 \cdot 10^{-45} \text{ cm}^2 c_h^2 \left(\frac{115 \text{ GeV}}{m_h} \right)^4 \left(\frac{700 \text{ GeV}}{f} \right)^4 \left(\frac{m_\chi}{100 \text{ GeV}} \right)^2 \left(\frac{\mu_\chi}{1 \text{ GeV}} \right)^2 \left(\frac{\lambda_N}{0.5} \right)^2$$

$$\sigma_{\text{SI}}^{\text{glue}} \approx 2 \cdot 10^{-48} \text{ cm}^2 \left(\frac{700 \text{ GeV}}{M_\lambda} \right)^2 \left(\frac{700 \text{ GeV}}{f} \right)^4 \left(\frac{N_\psi}{5} \right)^4 \left(\frac{q_\psi}{2} \right)^4 \left(\frac{\mu}{1 \text{ GeV}} \right)^2$$

using $c_{an} = \alpha_s q_\psi N_\psi / 8\pi$

Why are the $\chi\chi \rightarrow aa$ annihilations p -wave?

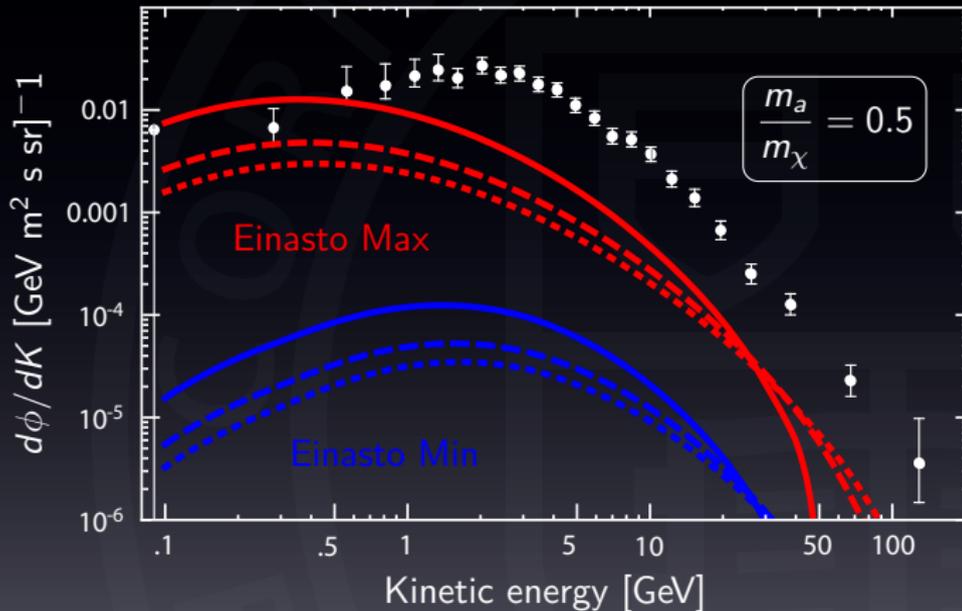
If the initial state is a particle-antiparticle pair with zero total angular momentum and the final state is CP even, then the process must vanish when $\nu = 0$.

Under CP a particle/antiparticle pair picks up a phase $(-)^{L+1}$. When $\nu = 0$ momenta are invariant and thus the initial state gets an overall minus sign. Since final state is CP even, the amplitude must vanish in this limit. For Dirac particles P is sufficient, but for Majorana particles CP is the well-defined operation.

This is why $\chi\chi \rightarrow G\tilde{G}$ is s -wave while $\chi\chi \rightarrow aa$ is p -wave.

Indirect detection: \bar{p} flux vs. PAMELA

$$f = 700 \text{ GeV}, Q_\Psi = 2, \delta = \frac{3}{2}, N_\Psi = 5$$



Dotted: $m_\chi = 150 \text{ GeV}, b_1 = 1$
Dashed: $m_\chi = 100 \text{ GeV}, b_1 = 1.5$
Solid: $m_\chi = 50 \text{ GeV}, b_1 = 3$

Using Einasto DM Halo profile in 1012.4535, 1009.0224

Indirect detection: Fermi-LAT

γ -ray line search: 30 – 200 GeV

- Upper bound $\langle\sigma v\rangle_{\gamma\gamma} < 2.5 \times 10^{-27} \text{ cm}^3/\text{s}$
- $\chi\chi \rightarrow a \rightarrow \gamma\gamma$ via anomaly
- For SU(5) fundamentals, $\langle\sigma v\rangle_{\gamma\gamma} \sim 2 \times 10^{-3} \langle\sigma v\rangle_{gg}$
- $\mathcal{O}(10)$ smaller than bound even for extreme parameters

Diffuse γ -ray spectrum: 20 – 100 GeV

- Bounds $\chi\chi$ to charged particles, π^0 s
- $\chi\chi \rightarrow a \rightarrow gg$ via anomaly
- $\mathcal{O}(10)$ smaller than bound

Photo-production from DM annihilation: spheroidal galaxies

- Low mass DM $m_\chi \lesssim 60 \text{ GeV}$, constrains bb decays
- GF: annihilation σ always at least a factor of 3 lower

<http://fermi.gsfc.nasa.gov/science/symposium/2011/program>

Collider production

Collider production through gluons. **ISR monojet** signature is sensitive to $\sigma_{SI}^N \sim 10^{-46} \text{ cm}^2$ at the LHC with 100 fb^{-1} .

The dim-7 anomaly operators are too small:

$$\mathcal{L} \supset -\frac{C_{an}^2}{2M_\chi f^2} \bar{\chi} \chi G G - \frac{i C_{an}^2}{2M_\chi f^2} \bar{\chi} \gamma^5 \chi G \tilde{G}$$

$gg \rightarrow a^* \rightarrow \chi\chi$ may be within 5σ reach with 100 fb^{-1}

1005.1286, 1005.3797, 1008.1783, 1103.0240, 1108.1196

Cascade decays: $\text{LOSP} \rightarrow \chi$ through

- $\bar{\chi} G \lambda$ anomaly
- $\chi - \tilde{H}$ kinetic mixing

Decays typically prompt, a reconstruction is difficult for light masses.

Heavy fermions Ψ in anomaly may appear as “fourth generation” quarks

Nuclear matrix element and matching

The nucleon matrix element at vanishing momentum transfer:

$$M_N = \langle \Theta_{\mu}^{\mu} \rangle = \langle N | \sum_{i=u,d,s} m_i \bar{q}_i q_i + \frac{\beta(\alpha)}{4\alpha} G_{\alpha\beta}^a G_{\alpha\beta}^a | N \rangle$$

from: Shifman, Vainshtein, Zakharov. Phys. Lett 78B (1978)

$\beta = -9\alpha^2/2\pi + \dots$ contains only the light quark contribution, M_N is the nucleon mass. The GG matches onto the nucleon operator $\bar{N}N$.

$$M_N f_{i=u,d,s}^{(N)} = \langle N | m_i \bar{q}_i q_i | N \rangle \quad f_g^{(N)} = 1 - \sum_{i=u,d,s} f_i^{(N)}$$

Nuclear matrix element and matching

$$\frac{\beta(\alpha)}{4\alpha} G_{\alpha\beta}^a G_{\alpha\beta}^a \longrightarrow M_N \left(1 - \sum_{i=u,d,s} f_i^{(N)} \right) \bar{N} N$$

Where $f_{u,d}^{(N)} \ll f_s^{(N)} \approx 0.25$. For a detailed discussion, see 0801.3656 and 0803.2360.

Image Credits and Colophon

- 'Zombie arm' illustration from <http://plantsvszombies.wikia.com>
- Beamer theme **Flip**, available online <http://www.lepp.cornell.edu/~pt267/docs.html>
- All other images were made by Flip using TikZ and Illustrator